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Interaction effects and dimensional crossover in disordered tunnelling superlattices

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Abstract. Effects of mutual interactions on the conductivity tensor and the density of states have been studied in the weakly localized regime of disordered tunnelling superlattices. The analytical results for the interaction corrections to both quantities are obtained as functions of the interlayer coupling t and the temperature T. We show that the temperature dependences of both corrections change from three-dimensional to two-dimensional behaviour with decreasing t, and that the dimensional crossover occurs at $t \sim 1/\tau_0$, with τ_0 the relaxation time due to impurity scattering.

1. Introduction

In recent years considerable progress has been made in understanding the nature of electronic states in disordered systems, especially disordered metals. The scaling theory has been successfully applied to the localization problem [1–3]. One of the important results of the scaling theory is that, in a two-dimensional (2D) system, there are no true metallic states, no matter how small the disorder is. The conductance decreases either logarithmically (weak localization) or exponentially (strong localization) when the size of the system is scaled down. By contrast, in a three-dimensional (3D) system, there is a metal–insulator transition on changing from weak localization to strong localization. One of the characteristics of weak localization is the decreasing conductivity with decreasing temperature. In the region of $T \ll 1/\tau_0$, with τ_0 the relaxation time due to impurity scattering, the localization contributions to conductivities in 2D and 3D systems vary directly as $\ln T$ and $T^{p/2}$, respectively. The value of p is usually between 1 (electron–electron scattering) and 2 (electron–phonon scattering).

Another disorder-related quantum correction to the conductivity has been proposed based on a different theory involving electron–electron correlations [4]. The interaction theory [4–7] has revealed that there exists an essential interplay between mutual interaction and randomness in disordered systems. As pointed out by Fukuyama [8], there exist two different diffusive processes that have very important influences on transport properties. One is the particle–particle diffusive process. It can be diagrammatically represented by a maximally crossed diagram, and is often called a Cooperon. The other is the particle–hole diffusive process. It is diagrammatically represented by a ladder diagram, and is called a diffuson. The Cooperon responsible for weak localization effects describes the interference effects between Bloch waves, and is very sensitive to perturbations that destroy the time-reversal symmetry. By contrast, the diffuson is directly related to the density–density correlation function, and is insensitive to perturbations that destroy the time-reversal symmetry. Both

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the Cooperon and the diffuson would be coupled with the dynamical scattering due to mutual interactions and lead to singular corrections to the density of states, the conductivity and other physical quantities. The diffuson will play a dominant role if the inverse screening radius, κ, of the mutual interaction is much smaller than $2k_F$, with k_F the Fermi momentum, i.e. $\kappa \ll 2k_F$. One of the important conclusions of the interaction theory is that the temperature dependences of the interaction corrections to both the conductivity and the density of states behave as $\ln T$ and $\sqrt{T}$ for 2D and 3D systems, respectively, as long as the condition $T \ll 1/\tau_0$ is satisfied.

There already exist some theoretical works concerned with the weak localization effects in anisotropic systems [9–11]. An important conclusion is that the scaling function and critical behaviour near the localization transition are unaffected by anisotropy, i.e.

$$\delta\sigma_\mu/\sigma_\mu = \delta\sigma_\nu/\sigma_\nu \tag{1}$$

where σ_μ and σ_ν are the Boltzmann conductivity contributions along the μ and ν directions, respectively, and $\delta\sigma_\mu$ and $\delta\sigma_\nu$ are the corresponding localization parts. Superlattices with layered structure are highly anisotropic systems. Szott *et al* [12] have calculated the weak-localization correction to conductivity in the low-frequency limit for tunnelling superlattices and concluded that, even though the effective-mass approximation along the growth direction might not be valid, equation (1) would hold. In contrast to the conclusion of Szott *et al*, Lu and Horing [13] pointed out that the scaling relation (1) holds only for systems with an anisotropic effective mass along differing directions, and it does not hold for tunnelling superlattices with small miniband width.

Up to now, anisotropic effects of mutual interactions for disordered superlattices have not been taken into account. They are expected to have important influences on the transport properties. In this paper, following Fukuyama [6,7], we explore in detail the effects of mutual interactions for disordered tunnelling superlattices under the condition $\kappa \gg 2k_F$, in which both the particle–hole and the particle–particle diffusive processes should be considered. We have obtained analytical expressions for the interaction corrections to the conductivity and the density of states to the lowest order of mutual interaction. Their temperature dependences are found to change from 3D to 2D behaviour with decreasing miniband width.

This paper is organized as follows. In section 2, we will present the model Hamiltonian with mutual interactions and the Boltzmann conductivity for a disordered tunnelling superlattice. The evaluations for the interaction corrections to the self-energy and the density of states will be presented in section 3. We will, in section 4, calculate and discuss the interaction correction to the conductivity. Finally, a brief summary is given in section 5.

2. The model for a tunnelling superlattice

Let us consider a disordered tunnelling superlattice, the miniband of which is described by a tight-binding model. The Hamiltonian for this system has the form:

$$H = \sum_{k,\sigma} \epsilon_k c^+_{k\sigma} c_{k\sigma} + U \sum_j \sum_{kq\sigma} \exp(-\mathrm{i}q\cdot R_j) c^+_{k+q,\sigma} c_{k\sigma} + \tfrac{1}{2} \sum_{kk'q} \sum_{\sigma\sigma'} v(q) c^+_{k+q,\sigma} c^+_{k'-q,\sigma'} c_{k'\sigma'} c_{k\sigma} \tag{2}$$

where the band energy is given by

$$\epsilon_k = (k_\parallel^2/2m) + t[1 - \cos(k_z a)]. \tag{3}$$

In equation (2), the first term is the kinetic energy of the electrons, the second one represents the interactions with normal impurities, and the last one stands for the interactions between electrons. U is the strength of the impurity potential whose force range is assumed to be point like. The impurities are located randomly and R_j is the impurity site. In equation (3), $k_\parallel$ and k_z are the wavevectors along the planar and z directions, respectively, a is the superlattice period, t is the interlayer coupling, and so $2t$ is the width of the superlattice miniband. In this work, since the Fermi energy is assumed to be much larger than the bandwidth, i.e. $\epsilon_F \gg 2t$, the density of states at the Fermi energy per spin is $N_1 = m/(2\pi a)$ and the density of charge carriers is given by $n = m\epsilon_F/(\pi a)$ [12]. If the concentration of impurities n_i is so small that $\epsilon_F \gg (2\tau_0)^{-1}$, we can take the Born approximation and get $(2\tau_0)^{-1} = \pi n_i U^2 N_1$.

According to the well known Kubo formula, in the absence of mutual interactions, the Boltzmann DC conductivity tensors can be easily calculated and are given by [12] $\sigma_\parallel = 2e^2 N_1 D_\parallel$ and $\sigma_z = 2e^2 N_1 D_z$, where $D_\parallel = \epsilon_F \tau_0/m$ and $D_z = t^2 a^2 \tau_0/2$ are the diffusion constants along the planar and z directions, respectively.

3. Interaction corrections to the self-energy and the density of states

We first study the diffuson and the Cooperon in an anisotropic disordered system. They are diagrammatically represented as in figures 1(a) and (b), respectively. The dashed lines with crosses represent the averaging procedure over the configuration of impurities, whose average concentration is n_i. The solid lines are the Green functions given by

$$G(k, i\epsilon_n) = [i\epsilon_n - \xi_k + i(2\tau_0)^{-1}\mathrm{sgn}\,\epsilon_n]^{-1} \tag{4}$$

where $\xi_k = \epsilon_k - \epsilon_F$, and $i\epsilon_n = i(2n+1)\pi T$ is the Matsubara frequency.

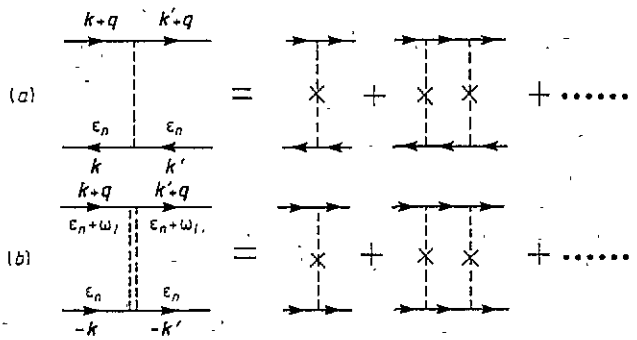

Figure 1. (a) Diffuson; (b) Cooperon.

For an isotropic disordered system, Fukuyama [7] has derived explicit expressions for the diffuson and the Cooperon. Using a similar procedure, we can calculate the functions for them for a tunnelling superlattice with an arbitrary band structure. The series of ladder diagrams shown in figure 1(a) is summed as

$$D(q, i\omega_l) = n_i U^2 [1 - n_i U^2 \Lambda(q, i\omega_l)]^{-1} \tag{5}$$

where $\Lambda(q, i\omega_l)$ is defined by

$$\Lambda(q, i\omega_l) = \sum_k G(k+q, i\epsilon_n + i\omega_l)G(k, i\epsilon_n) \tag{6}$$

and $\omega_l = 2l\pi T$. Since both $D_\mu q_\mu^2$ ($\mu = x, y, z$) and $|\omega_l|$ are assumed to be much smaller than $1/\tau_0$, when the product of two Green functions on the right-hand side of equation (6) is expanded as a power series of q and $|\omega_l|$, only their lowest-order terms need to be kept. We first consider the case of $\epsilon_n < 0$ and $\epsilon_n + \omega_l > 0$. In this case, substituting equation (4) into equation (6), and replacing the summation over k in equation (6) by the integral $\int N_1 \, d\xi_k$, we obtain

$$\Lambda(q, i\omega_l) = \frac{N_1}{4\pi} \int d\xi_k \int d\Omega \left(i\epsilon_n - \xi_k - \frac{i}{2\tau_0} \right)^{-1} \left[\left(i\epsilon_n - \xi_k + \frac{i}{2\tau_0} \right)^{-1} + i|\omega_l| \left(i\epsilon_n - \xi_k + \frac{i}{2\tau_0} \right)^{-2} + [v(k) \cdot q]^2 \left(i\epsilon_n - \xi_k + \frac{i}{2\tau_0} \right)^{-3} \right] \tag{7}$$

where $v(k) = \partial\epsilon_k/\partial k$ is the velocity for the electron in the state k, and the integration with respect to Ω is over the angle between $v(k)$ and the z axis. Equation (7) is easily evaluated, yielding

$$\Lambda(q, i\omega_l) = 2\pi\tau_0 N_1 (1 - |\omega_l|\tau_0 - D_\mu q_\mu^2 \tau_0) \tag{8}$$

where $D_\mu q_\mu^2 = D_\parallel q_\parallel^2 + D_z q_z^2$. Substituting equation (8) into equation (5) and using the relation $(2\tau_0)^{-1} = \pi n_i U^2 N_1$, we get the final expression for the diffuson as

$$D(q, i\omega_l) = [2\pi N_1 \tau_0^2 (D_\mu q_\mu^2 + |\omega_l|)]^{-1}. \tag{9}$$

It is easily shown from a similar derivation that, in the case of $\epsilon_n > 0$ and $\omega_l + \epsilon_n < 0$, the same result as equation (9) holds; but in the case of $\epsilon_n(\epsilon_n + \omega_l) > 0$, the result is quite different, $\Lambda(q, i\omega_l) = 0$, and so $D(q, i\omega_l) = (2\pi N_1 \tau_0)^{-1}$.

Following the derivation as above, we can obtain the expression for the Cooperon as shown in figure 1(*b*):

$$C(q, i\omega_l) = [2\pi N_1 \tau_0^2 (D_\mu q_\mu^2 + |\omega_l| + \tau_\varphi^{-1})]^{-1} \qquad \text{for } \epsilon_n(\epsilon_n + \omega_l) < 0. \tag{10}$$

Here τ_φ is the dephasing time. If the diffusion constant is isotropic, $D_\mu = D$ for any μ, equations (9) and (10) reduce to the isotropic results obtained by Fukuyama [7]. It should be emphasized that our derivation above does not depend on any special band structure. So, we have demonstrated that both equations (9) and (10) are suitable for arbitrary dispersion relations provided that the condition $\epsilon_n(\epsilon_n + \omega_l) < 0$ is satisfied.

We now calculate the interaction correction to the self-energy. In disordered metals, the mutual interaction $v(q)$ is dynamically screened and changed to the effective interaction $v(q, i\omega_l)$. For a clean system the lowest-order contributions in interaction with the self-energy are given by the well known Hartree–Fock processes. In the presence of randomness, however, these Hartree–Fock processes are coupled with diffusons and Cooperons, and the self-energy corrections due to the mutual interactions are given by figure 2 in [6], the corresponding effective interaction constants being parametrized as g_1, g_2, g_3 and g_4, respectively [6]. As an example, the g_1 processes (see figure 2) represent the Fock processes

coupled with the diffusons, and the contribution to the self-energy from figure 2(*a*) can be evaluated as follows:

$$\delta\Sigma_{1a} = -T\sum_{\omega_l}\sum_{k'k''q} v(q, i\omega_l)D^2(q, i\omega_l)G(k+q, i\epsilon_n + i\omega_l)G(k', i\epsilon_n)$$
$$\times G(k'+q, i\epsilon_n + i\omega_l)G(k'', i\epsilon_n)G(k''+q, i\epsilon_n + i\omega_l). \tag{11}$$

Substituting equations (4) and (9) into equation (11), and taking the approximation $G(k+q, i\epsilon_n + i\omega_l) \simeq -2i\tau_0\,\mathrm{sgn}(\epsilon_n + \omega_l)$, we replace the summations over k' and k'' by the integrals $\int N_1\,d\xi_{k'}$ and $\int N_1\,d\xi_{k''}$, respectively, and get

$$\frac{\delta\Sigma_{1a}}{\Sigma_0} = \frac{4Tg_1}{N_1}\sum_{\omega_l}(2\pi)^{-3}\int_0^{1/l_\parallel} 2\pi q_\parallel\,dq_\parallel \int_{-\pi/a}^{\pi/a} dq_z\,(D_\parallel q_\parallel^2 + D_z q_z^2 + |\omega_l|)^2$$
$$= \frac{Tg_1}{\pi^2 N_1 D_\parallel D_z^{1/2}}\sum_{\omega_l}|\omega_l|^{-1/2}\tan^{-1}\left[\frac{\pi}{a}\left(\frac{D_z}{|\omega_l|}\right)^{1/2}\right] \tag{12}$$

where $\Sigma_0 = -i(2\tau_0)^{-1}\mathrm{sgn}(\epsilon_n)$ is the self-energy due to impurity scattering in the absence of interactions, and $l_\parallel = (D_\parallel\tau_0)^{1/2}$ is the mean free path along the planar direction. The summation over ω_l is from $2\pi T$ to $1/\tau_0$. In the region of $T \ll 1/\tau_0$, the summation in equation (12) can be replaced by integration over ω, yielding

$$\frac{\Sigma_{1a}}{\Sigma_0} = \frac{2g_1}{4\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}} F(t\tau_0, T\tau_0) \tag{13}$$

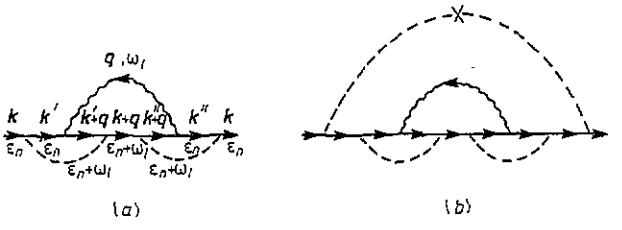

Figure 2. Interaction corrections to the self-energy due to g_1 processes.

where the function $F(x, y)$ is given by

$$F(x, y) = \frac{2}{\pi}\left\{\tan^{-1}\left(\frac{\pi x}{\sqrt{2}}\right) - (2\pi y)^{1/2}\tan^{-1}\left[\left(\frac{\pi x^2}{4y}\right)^{1/2}\right]\right\}$$
$$+ \frac{x}{\sqrt{2}}[\ln(2+\pi^2x^2) - \ln(4\pi y + \pi^2x^2)]. \tag{14}$$

Since the evaluation of the contribution for figure 2(*b*) is very similar to that for figure 2(*a*), we simply write down its expression:

$$\frac{\Sigma_{1b}}{\Sigma_0} = -\frac{g_1}{4\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}} F(t\tau_0, T\tau_0). \tag{15}$$

Summing equations (13) and (15), we obtain the interaction correction to the self-energy due to g_1 processes, $\delta\Sigma_1 = \delta\Sigma_{1a} + \delta\Sigma_{1b}$, as follows:

$$\frac{\delta\Sigma_1}{\Sigma_0} = \frac{g_1}{4\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}} F(t\tau_0, T\tau_0). \tag{16}$$

Cooperons are coupled with the interactions as shown in g_2 and g_4 processes. Their contribution will vanish in the region of $T \ll 1/\tau_\varphi$, where τ_φ comes from perturbations that destroy the time-reversal symmetry. In the case of $1/\tau_\varphi \ll T \ll 1/\tau_0$, calculations similar to that for g_1 processes give the contributions due to g_i processes ($i = 2, 3, 4$):

$$\frac{\delta\Sigma_2}{\Sigma_0} = \frac{g_2}{4\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}} F(t\tau_0, T\tau_0)$$

$$\frac{\delta\Sigma_3}{\Sigma_0} = \frac{-2g_3}{4\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}} F(t\tau_0, T\tau_0)$$

$$\frac{\delta\Sigma_4}{\Sigma_0} = \frac{-2g_4}{4\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}} F(t\tau_0, T\tau_0).$$

By summing these $\delta\Sigma_i/\Sigma_0$, the total interaction correction to the self-energy is given by

$$\frac{\delta\Sigma}{\Sigma_0} = \frac{g}{4\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}} F(t\tau_0, T\tau_0) \tag{17}$$

where $g = g_1 + g_2 - 2(g_3 + g_4)$ with the factor -2 coming from the spin degeneracy in the Hartree processes of our spin-independent interactions. If we define $\Sigma = \Sigma_0 + \delta\Sigma = -\mathrm{i}(2\tau)^{-1}\mathrm{sgn}(\epsilon_n)$, the renormalized scattering rate due to mutual interactions can be obtained as

$$\frac{1}{\tau} = \frac{1}{\tau_0}\left(1 + \frac{g}{4\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}} F(t\tau_0, T\tau_0)\right). \tag{18}$$

The quantum correction to the density of states at the Fermi surface, $\delta N/N_0$, due to interactions is straightforwardly obtained from a knowledge of $\delta\Sigma/\Sigma_0$ through the relation

$$\delta N/N_0 = -\delta\Sigma/\Sigma_0. \tag{19}$$

This equation has been shown to hold for isotropic systems [7]. Here we will show that equation (19) is a general relation in the lowest approximation and it still holds for anisotropic layered systems. To see clearly the relationship between $\delta N/N_0$ and $\delta\Sigma/\Sigma_0$, we start from the definition of the density of states at the Fermi surface,

$$N = -\frac{2}{\pi}\,\mathrm{Im}\sum_k G^{\mathrm{R}}(k, 0) \tag{20}$$

where $G^{\mathrm{R}}(k, 0) = (-\xi_k - \Sigma^{\mathrm{R}})^{-1}$ is the interaction-correction retarded Green function, $\Sigma^{\mathrm{R}} = -\mathrm{i}(2\tau)^{-1}$ is the interaction-correction retarded self-energy, and

$$\mathrm{Im}\, G^{\mathrm{R}}(k, 0) = \frac{\mathrm{Im}\,\Sigma^{\mathrm{R}}}{\xi_k^2 + (\mathrm{Im}\,\Sigma^{\mathrm{R}})^2} = \frac{(\mathrm{Im}\,\Sigma_0^{\mathrm{R}})(1 + \delta\Sigma/\Sigma_0)}{\xi_k^2 + (\mathrm{Im}\,\Sigma_0^{\mathrm{R}})^2(1 + \delta\Sigma/\Sigma_0)^2} \tag{21}$$

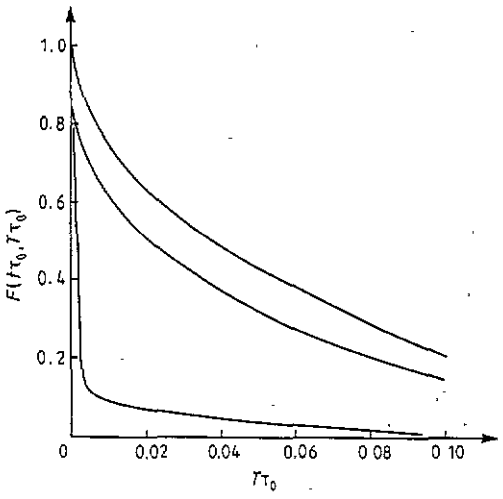

Figure 3. The temperature dependences of the function $F(t\tau_0, T\tau_0)$ for several values of $t\tau_0$. For $t\tau_0 = 0.05$, the lowest curve shows logarithmic behaviour, while for $t\tau_0 = 10$ the top curve is square-root-like, corresponding to 2D and 3D behaviour, respectively. The middle curve corresponds to $t\tau_0 = 1$.

with $\Sigma_0^R = -i(2\tau_0)^{-1}$ being the retarded self-energy due to impurity scattering. The expansion of equation (21) to the lowest-order approximation in $\delta\Sigma/\Sigma_0$ is

$$\operatorname{Im} G^R(k,0) = \frac{\operatorname{Im}\Sigma_0^R}{\xi_k^2 + (\operatorname{Im}\Sigma_0^R)^2}\left(1 + \frac{\delta\Sigma}{\Sigma_0} - \frac{2(\operatorname{Im}\Sigma_0^R)^2}{\xi_k^2 + (\operatorname{Im}\Sigma_0^R)^2}\frac{\delta\Sigma}{\Sigma_0}\right). \tag{22}$$

Substituting equation (22) into equation (20), and replacing the summation over k in equation (20) by the integral $\int N_1\, d\xi_k$, we finally obtain

$$N = N_0(1 - \delta\Sigma/\Sigma_0) \qquad \text{or} \qquad \delta N/N_0 = -\delta\Sigma/\Sigma_0$$

where $N_0 = 2N_1$, and we have used

$$\lim_{\operatorname{Im}\Sigma_0^R \to 0} \frac{1}{\xi_k^2 + (\operatorname{Im}\Sigma_0^R)^2} = \frac{\pi}{|\operatorname{Im}\Sigma_0^R|}\delta(\xi_k)$$

and

$$\lim_{\operatorname{Im}\Sigma_0^R \to 0} \frac{1}{[\xi_k^2 + (\operatorname{Im}\Sigma_0^R)^2]^2} = \frac{\pi}{2(\operatorname{Im}\Sigma_0^R)^2}\delta(\xi_k).$$

It is worth emphasizing that the derivation above does not also depend on the energy band structure, so that equation (19) is a general relation.

Now, we have obtained the analytical expressions for the interaction corrections to the self-energy and the density of states at the Fermi surface for a disordered tunnelling superlattice. They depend on the interlayer coupling t and the temperature T. In order to compare our results with that of 2D and 3D systems, we take the following limits of the function $F(x, y)$:

$$F(x,y) = \begin{cases} 1 - (2\pi y)^{1/2} & (x \gg 1,\ y \ll 1) \\ -(x/\sqrt{2})\ln(2\pi y) & (x^2 \ll y \ll 1). \end{cases} \tag{23}$$

The function $F(t\tau_0, T\tau_0)$ is shown in figure 3. From equation (23), we can see that, when the interlayer coupling t is so large that $t \gg 1/\tau_0$, the first limiting result corresponds to anisotropic 3D behaviour; while when t is small enough so that $t \ll (T/\tau_0)^{1/2} \ll 1/\tau_0$, the second limiting result corresponds to isotropic 2D behaviour. Therefore the interaction corrections to self-energy and density of states at the Fermi surface behave as 3D (proportional to $T^{1/2}$), changing to 2D (proportional to $\ln T$) when the bandwidth $2t$ decreases from $t \gg 1/\tau_0$ to $t \ll (T/\tau_0)^{1/2}$, and a dimensional crossover occurs at $t \sim 1/\tau_0$.

4. Interaction correction to the conductivity

The conductivity tensors are determined by the well known Kubo formula

$$\sigma_{\alpha\beta}(\omega) = -\frac{e^2}{\omega}\,\mathrm{Im}\left(\sum_{k,k'} v_\alpha(k)v_\beta(k')\Pi(k,k';\mathrm{i}\omega_\lambda)_{\mathrm{i}\omega_\lambda\to\omega+\mathrm{i}0^+}\right) \tag{24}$$

where ω is the frequency of the applied electric field, $\omega_\lambda = 2\lambda\pi T$, and $\Pi(k,k';\mathrm{i}\omega_\lambda)$ is the polarization part due to mutual interaction and impurity scattering. The interaction corrections to the conductivities due to g_i processes ($i = 1,2,3,4$) have been given by figures 4–9 in [6]. For the anisotropic layered systems under consideration, we have calculated contributions from these processes, and the evaluations are very elaborate and lengthy. As an example, in the appendix, we will evaluate only the contribution from one of the g_1 processes. For the contributions from other processes including g_i ($i = 1,2,3,4$), since their derivations are somewhat similar to that given in the appendix, we simply write down their expressions as follows:

$$\delta\sigma_\parallel^1/\sigma_\parallel = -6g_1B_\parallel \qquad \delta\sigma_z^1/\sigma_z = -12g_1B_z \tag{25a}$$

$$\delta\sigma_\parallel^2/\sigma_\parallel = -8g_2A + 18g_2C_\parallel \qquad \delta\sigma_z^2/\sigma_z = -8g_2A + 36g_2C_z \tag{25b}$$

$$\delta\sigma_\parallel^3/\sigma_\parallel = 12g_3B_\parallel \qquad \delta\sigma_z^3/\sigma_z = 24g_3B_z \tag{25c}$$

$$\delta\sigma_\parallel^4/\sigma_\parallel = 16g_4A - 36g_4C_\parallel \qquad \delta\sigma_z^4/\sigma_z = 16g_4A - 72g_4C_z \tag{25d}$$

where

$$A = \frac{4\pi Ta}{m}\sum_{\omega_l>0}\sum_q \omega_l(D_\mu q_\mu^2 + \omega_l + \tau_\varphi^{-1})^{-3} \tag{26a}$$

$$B_\parallel = \frac{4\pi Ta}{m}\sum_{\omega_l>0}\sum_q D_\parallel q_\parallel^2\omega_l(D_\mu q_\mu^2 + \omega_l)^{-4} \tag{26b}$$

$$B_z = \frac{4\pi Ta}{m}\sum_{\omega_l>0}\sum_q D_z q_z^2\omega_l(D_\mu q_\mu^2 + \omega_l)^{-4} \tag{26c}$$

$$C_\parallel = \frac{4\pi Ta}{m}\sum_{\omega_l>0}\sum_q D_\parallel q_\parallel^2\omega_l(D_\mu q_\mu^2 + \omega_l + \tau_\varphi^{-1})^{-4} \tag{26d}$$

$$C_z = \frac{4\pi Ta}{m}\sum_{\omega_l>0}\sum_q D_z q_z^2\omega_l(D_\mu q_\mu^2 + \omega_l + \tau_\varphi^{-1})^{-4}. \tag{26e}$$

In the region of $1/\tau_\varphi \ll T \ll 1/\tau_0$, all the summations over ω_l and q in equations (26*a*)–(26*e*) may be evaluated. We obtain

$$B_\parallel = C_\parallel = \frac{A}{3} = \frac{f(t\tau_0, T\tau_0)}{48\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}} \tag{27a}$$

$$B_z = C_z = \frac{\phi(t\tau_0, T\tau_0)}{96\pi^2 N_1 D_\parallel (D_z\tau_0)^{1/2}}. \tag{27b}$$

Here the functions $f(x, y)$ and $\phi(x, y)$ are given by

$$f(x,y)=\frac{2}{\pi}\left\{\tan^{-1}\left(\frac{\pi x}{\sqrt{2}}\right)-(2\pi y)^{1/2}\tan^{-1}\left[\left(\frac{\pi x^2}{4y}\right)^{1/2}\right]\right\}$$
$$+\sqrt{2}x[\ln(2+\pi^2x^2)-\ln(4\pi y+\pi^2x^2)] \tag{28}$$

$$\phi(x,y)=\frac{2}{\pi}\left\{\tan^{-1}\left(\frac{\pi x}{\sqrt{2}}\right)-(2\pi y)^{1/2}\tan^{-1}\left[\left(\frac{\pi x^2}{4y}\right)^{1/2}\right]\right\}$$
$$+\sqrt{2}\pi^2x^3[(\pi^2x^2+4\pi y)^{-1}-(\pi^2x^2+2)^{-1}]. \tag{29}$$

Substituting equations (27*a*) and (27*b*) into equations (25*a*)–(25*d*), we get the total corrections to the conductivities, $\delta\sigma_\parallel=\sum_{i=1}^{4}\delta\sigma_\parallel^i$ and $\delta\sigma_z=\sum_{i=1}^{4}\delta\sigma_z^i$:

$$\frac{\delta\sigma_\parallel}{\sigma_\parallel}=-\frac{g}{8\pi^2N_1D_\parallel(D_z\tau_0)^{1/2}}f(t\tau_0,T\tau_0) \tag{30a}$$

$$\frac{\delta\sigma_z}{\sigma_z}=-\frac{g}{8\pi^2N_1D_\parallel(D_z\tau_0)^{1/2}}\phi(t\tau_0,T\tau_0)-\frac{g_2-2g_4}{2\pi^2N_1D_\parallel(D_z\tau_0)^{1/2}}[f(t\tau_0,T\tau_0)-\phi(t\tau_0,T\tau_0)]. \tag{30b}$$

Equations (30*a*) and (30*b*) are the main results we obtain in this section. They are general expressions for the interaction corrections to anisotropic conductivities in disordered tunnelling superlattices provided that the band theory and the Bloch–Boltzmann picture hold true. So, it is expected that in the two limiting cases of $t\tau_0\gg1$ and $t\tau_0\ll1$, equations (30*a*) and (30*b*) should reduce to the well known forms for 3D and 2D systems, respectively. To verify this point, it is necessary to discuss the asymptotic behaviour of functions $f(x, y)$ and $\phi(x, y)$ given in equations (28) and (29). First, in the limit where $x\gg1$, together with $y\ll1$, we have

$$f(x,y)=\phi(x,y)=1-(2\pi y)^{1/2} \tag{31}$$

which is independent of x. Secondly, in the opposite limit where $x^2\ll y\ll1$, we have

$$f(x,y)=-\sqrt{2}\,x\ln(2\pi y) \tag{32}$$
$$\phi(x,y)=\sqrt{2}\,\pi x^3/(3y)\ll f(x,y). \tag{33}$$

With the aid of equations (31)–(33), the expressions for $\delta\sigma_\parallel/\sigma_\parallel$ and $\delta\sigma_z/\sigma_z$ become

$$\frac{\delta\sigma_\parallel}{\sigma_\parallel}=\frac{\delta\sigma_z}{\sigma_z}=\frac{g}{8\pi^2N_1D_\parallel(D_z\tau_0)^{1/2}}[(2\pi T\tau_0)^{1/2}-1]\qquad\text{for } T\ll1/\tau_0\ll t \tag{34}$$

and

$$\frac{\delta\sigma_\parallel}{\sigma_\parallel}=\frac{g}{4\pi^2N_1aD_\parallel}\ln(2\pi T\tau_0)$$
$$\frac{\delta\sigma_z}{\sigma_z}=\frac{g_2-2g_4}{\pi^2N_1aD_\parallel}\ln(2\pi T\tau_0)\qquad\text{for } t\ll(T/\tau_0)^{1/2}\ll1/\tau_0. \tag{35}$$

For large t ($t\tau_0 \gg 1$) the system can be regarded as an anisotropic 3D system. In this limit the interaction correction to the conductivities given in equation (34) is proportional to $(2\pi T\tau_0)^{1/2} - 1$, which is just the familiar behaviour of a 3D system. Moreover, the scaling relation (1) holds in this case, indicating that for anisotropic 3D systems the scaling theory of localization and the interaction theory have the same conclusion that the scaling behaviour is unaffected by anisotropy. The latter is a new result in the present paper, while the former is well known. On the other hand, for small t ($t\tau_0 \ll 1$) the tunnelling superlattice becomes a quasi-2D system, and equation (35) yields $\delta\sigma_\parallel/\sigma_\parallel \propto \ln(2\pi T\tau_0)$, which is characteristic of 2D behaviour. It is easy to see that in this case the scaling relation (1) no longer holds. Its invalidity may stem from the consideration that when the interlayer coupling t is small enough, the Bloch–Boltzmann picture breaks down for the out-of-plane conduction. Finally, in the intermediate region of t between the two limits, the analytical expressions (30*a*) and (30*b*) for $\delta\sigma_\parallel$ and $\delta\sigma_z$ describe the crossover behaviour of their temperature dependences from 3D to 2D systems with decreasing t. It is worth mentioning that in this transition region from 3D to 2D behaviour the scaling relation (1) does not seem to hold. This indicates that the conclusion that the scaling behaviour is unaffected by anisotropy is suitable only for the limit of $t\tau_0 \gg 1$.

5. Summary

In this paper, to the lowest order of mutual interaction, we have calculated the quantum corrections to the self-energy, the density of states at the Fermi surface and the anisotropic conductivity tensor in a disordered tunnelling superlattice. The analytical expressions for these interaction corrections as functions of t and T are obtained for the first time. The temperature dependences of these corrections are shown to be dependent upon the interlayer coupling t and change from 3D to 2D behaviour with decreasing t.

In the weakly localized regime of isotropic disordered systems, the expansion parameter for the perturbation theory is $(\epsilon_F\tau_0)^{-1}$. However, in the present model, which may be highly anisotropic, the small parameter is chosen as $[N_1 D_\parallel (D_z\tau_0)^{1/2}]^{-1}$, which is proportional to $(\epsilon_F\tau_0)^{-1}(t\tau_0)^{-1}$, if $t \gtrsim 1/\tau_0$; while in the case of $t \ll (T/\tau_0)^{1/2} \ll 1/\tau_0$, the small parameter becomes $(N_1 a D_\parallel)^{-1}$, which is proportional to $(\epsilon_F\tau_0)^{-1}$.

Another conclusion of our work is that, in the interaction theory, the scaling relation (1) does not generally hold, but does hold only for the limit of $t \gg 1/\tau_0$, which is the same as in the weak-localization theory [13]. In both theories, the invalidity of equation (1) is apparent from the smallness of the miniband in the tunnelling superlattice where the anisotropic effective-mass approximation is not appropriate.

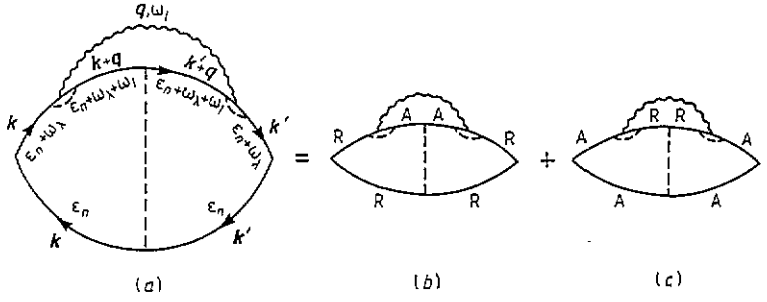

Figure 4. A diagram for conductivity among g_1 processes.

It is important to point out that, in the extreme 2D limit, the mean free path for motion along the z direction $l_z \sim t\tau_0 a$ is much less than the lattice spacing and therefore much less than the Fermi wavelength, invalidating a quasiclassical treatment, which leads to the failure of Boltzmann transport theory in this limit. Another limitation of our theory is that the model of isotropic impurity scattering is not very realistic in the extreme 2D limit, when the scattering cross section for processes involving momenta k and k' parallel will surely be different from the ones involving momenta changing from parallel to perpendicular to the layers. Considering these limitations, some refined theory is necessary.

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Appendix

As an example the contribution to the conductivity from the process of figure 4(a) in [6], which is one of the g_1 processes, will be evaluated. The frequency and momentum dependences are explicitly shown in figure 4, where R and A represent the retarded and advanced Green functions, respectively, and $\omega_\lambda > 0$.

The polarization part $\Pi(k, k'; i\omega_\lambda)$ in this diagram is

$$\begin{aligned}\Pi(k, k'; i\omega_\lambda) = -2T^2 \sum_q \sum_{\epsilon_n \omega_\lambda} & \bar{v}(q, i\omega_l)\Gamma^2(q, i\omega_\lambda)D(q, i\omega_\lambda + i\omega_l) \\ & \times G(k, i\epsilon_n)G(k, i\epsilon_n + i\omega_\lambda)G(k+q, i\epsilon_n + i\omega_\lambda + i\omega_l) \\ & \times G(k', i\epsilon_n)G(k', i\epsilon_n + i\omega_\lambda)G(k'+q, i\epsilon_n + i\omega_\lambda + i\omega_l) \qquad \text{(A1)}\end{aligned}$$

where $\Gamma(q, i\omega_l)$ is the function for the interaction vertex, and is easily shown to be

$$\Gamma(q, i\omega_l) = [\tau_0(D_\mu q_\mu^2 + |\omega_l|)]^{-1}. \qquad \text{(A2)}$$

Substituting equations (9) and (A2) into equation (A1), we have

$$\begin{aligned}-\frac{e^2}{\omega} \sum_{k,k'} & v_x(k)v_x(k')\Pi(k, k'; i\omega_\lambda) \\ & = \frac{e^2 T^2 g_1}{\pi N_1^2 \tau_0^4 m^2 \omega} \sum_q \sum_{\epsilon_n \omega_\lambda} (D_\mu q_\mu^2 + |\omega_l|)^{-2}(D_\mu q_\mu^2 + |\omega_l + \omega_\lambda|)^{-1} \\ & \quad \times \sum_k k_x G(k, i\epsilon_n)G(k, i\epsilon_n + i\omega_\lambda)G(k+q, i\epsilon_n + i\omega_\lambda + i\omega_l) \\ & \quad \times \sum_{k'} k'_x G(Pk', i\epsilon_n)G(k', i\epsilon_n + i\omega_\lambda)G(k'+q, i\epsilon_n + i\omega_\lambda + i\omega_l). \qquad \text{(A3)}\end{aligned}$$

The condition $\epsilon_n(\epsilon_n + \omega_\lambda + \omega_l) < 0$ as well as $(\epsilon_n + \omega_\lambda)(\epsilon_n + \omega_\lambda + \omega_l) < 0$ leads to the following classification of the regions of ϵ_n and ω_l:

(i) $\omega_l < -\omega_\lambda$, $0 < \epsilon_n < -\omega_l - \omega_\lambda$,
(ii) $\omega_l > 0$, $-\omega_\lambda - \omega_l < \epsilon_n < -\omega_\lambda$.

They correspond to figures 4(*b*) and (*c*) respectively. In the summations over k and k' in equation (A3), by expanding in terms of q to the lowest order and replacing the summations over k and k' by the integrals $\int N_1 \, d\xi_k$ and $\int N_1 \, d\xi_{k'}$, respectively, we obtain

$$-\frac{e^2}{\omega}\sum_{k,k'} v_x(k)v_x(k')\Pi(k,k';i\omega_\lambda) = \frac{4\pi a g_1 T\sigma_\parallel}{m\omega}$$
$$\times \sum_q \Bigg(\sum_{\omega_l<-\omega_\lambda} \frac{-D_\parallel q_\parallel^2(\omega_l+\omega_\lambda)}{(D_\mu q_\mu^2+|\omega_l|)^2(D_\mu q_\mu^2+|\omega_l+\omega_\lambda|)}$$
$$+\sum_{\omega_l>0} \frac{-D_\parallel q_\parallel^2\omega_l}{(D_\mu q_\mu^2+|\omega_l|)^2(D_\mu q_\mu^2+|\omega_l+\omega_\lambda|)}\Bigg). \tag{A4}$$

We expand equation (A4) in terms of ω_λ, yielding

$$-\frac{e^2}{\omega}\sum_{k,k'} v_x(k)v_x(k')\Pi(k,k';i\omega_\lambda) = O(0) - \frac{12\pi a g_1 T\sigma_\parallel}{m\omega}$$
$$\times \sum_q \sum_{\omega_l>0} \frac{D_\parallel q_\parallel^2\omega_l}{(D_\mu q_\mu^2+|\omega_l|)^4}\omega_\lambda + O(\omega_\lambda^2) \tag{A5}$$

where $O(0)$ and $O(\omega_\lambda^2)$ represent the zeroth-order and second-order terms of ω_λ, respectively. Substituting equation (A5) into equation (24), we get the contribution to the planar conductivity from this diagram:

$$\delta\sigma_\parallel(\text{figure 4})/\sigma_\parallel = 3g_1 B_\parallel.$$

A similar evaluation leads to

$$\delta\sigma_z(\text{figure 4})/\sigma_z = 6g_1 B_z.$$

The expressions for $B_\parallel$ and B_z have been given in equations (26*b*) and (26*c*), respectively.

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